Calogerian models, osculation method and low-lying spectra of many-particle anharmonic oscillators

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Thirty five years ago, F. Calogero [1] revealed the exact solvability of a system of three particles on a line, bound by so "realistic" two-particle spiked harmonic-oscillator forces that his paper inspired a lasting interest in similar systems. Our group in NPI joined these research activities in 2001. We found that a new insight may be gained via the so called $\mathcal{PT}-$symmetric complexification of the two-body forces [2,3] and/or via the so called $1/L$ expansions [4].

We succeeded in producing further new results during the last two years. Firstly, on a methodical level, several alternative versions of the $\mathcal{PT}-$symmetric complexifications have been found and clarified [5]. In a way illustrated by the picture below (cf. ref. [6]) we also found that perturbation theory offers the most comfortable tool for switching between solvable models and their "more realistic" descendants.

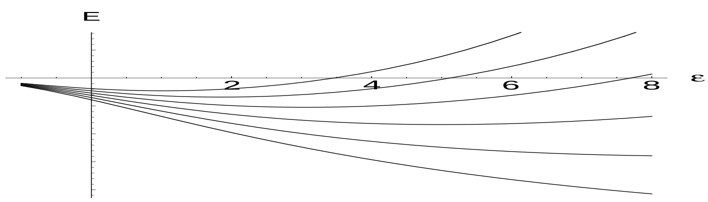

Fig. 1: *The characteristic $\varepsilon-$dependence of the six lowest energies in $V(r) \sim r^2(\mathrm{i}r)^\varepsilon$.*

Our main progress achieved in the years 2003 - 2004 concerns the fully Hermitian Calogerian models. In the most straightforward development we stayed in the exactly solvable regime and succeeded in a modification of the two- and three-body repulsive interactions [7].

In a temporary climax of our effort, both the attractive long-range two-body forces and their short-range singular parallels were allowed to be of an arbitrary integer power [8]. The exact solvability has been lost of course. Still, all the class of the new anharmonic models proved unexpectedly user-friendly and remains under our current investigation.

A mathematical key to our fresh results lies in a derivation of certain new trigonometric identities. In the solvable cases of ref. [7], these identities were not too complicated. In contrast, the mathematics related to the latter, phenomenologically motivated anharmonic many-body oscillators remained perturbative. In this context, "osculation" approximations in zero order proved particularly efficient. The proofs of the fairly complicated necessary trigonometric identities employ the computer-assisted symbolic algebraic manipulations.

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