Spectral theory enjoys finding connections between classical vibrations and quantum Schroedigner operators. A particularly popular class of illustrations involves the relation between oscillations of strings and various delta-function-type solvable descendants of the quantum Kronig-Penney model (cf. Kronig R de L and Penney W G 1931 Proc. R. Soc. A 130 499). A sample of these results is provided in the four-page paper in question, the subject of which is inspired by the incompleteness of equivalence between the spectra of the Krein's inhomogeneous strings (with positive masses – cf. refs. [5] - [7]) and derivative-delta-function quantum oscillators (without positivity constraint, i.e., mathematically more general and methodically more challenging). After a "warm-up" theorem 1 (showing that n derivative-delta-functions produce n energy levels), theorems 2 and 3 (on the same interaction but with respective support on the Cantor discontinuum and on an equidistant point set in the infintely-many-point limit) is more mind-boggling since the only limiting point of the "bound state energies" proves to be minus infinity.

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