This is a review text file submitted electronically to MR.

Reviewer: Znojil, Miloslav

Reviewer number: 13388

Address:
NPI AS CR
250 68 Rez
Czech Republic
znojil@ujf.cas.cz

Author: This line will be completed by the MR staff.

Short title: This line will be completed by the MR staff.

Control number: 1783953

Primary classification: 30E15

Secondary classification(s): 81Q15 30B50 40G99 34E20 41A60 65E05

Review text:

In quantum physics, people search for energies E computed as functions of a coupling g, mostly in a weak-coupling regime. In cubic case the textbook “perturbation” recipe uses just a simple power series in g and its re-summations. It works/fails for positive/negative arg(g), respectively. The paper shows that in the latter case a more complicated series ansatz is needed, comprising an exponentially quickly decreasing sequence of subseries in (both powers and logarithms of) g. Its form and the method of its construction are derived, showing a broad new domain of the applicability of perturbative considerations.

The idea of proof lies in a re-scaling of the coordinate, after which the coupling g re-appears in the role of a “fake” Planck constant h. The next WKB-like step starts near the two different turning points and searches for the unknown arguments in the respective parabolic-cylinder and Airy solutions. Via an ansatz of power series form used for these arguments one constructs them in the usual recurrent manner. The final step matches the two solutions and gives the final unusual asymptotic series for the energy in a wedge 0 ¿ arg(g) ¿ - const.

As an independent check of reliability of this matching method the usual perturbation series is re-obtained for arg(g) ¿ 0. For arg(g) ¡ 0 the matching is “twice as difficult” but its ability to produce the series (to an arbitrary order in both the scales of smallness) remains unchanged. The result is presented up to the third exponential order. The absence of rigorous proofs of certain intermediate steps [cf. eq. (19)] is compensated by persuasive numerical checks [showing, e.g., the validity of eq. (19) in the first ten orders]. Similarly, the main hypothesis of convergence of the final re-summation is supported by the reproduction of results of a brute force single-precision numerical computation by the present double-series approximants in the (4 times 28)-th order.

Important paper. After the Harrel's logarithms-containing series for energies in singular potentials as obtained in the late seventies, another very explicit and instructive sample of non-polynomial asymptotic approximants with an immediate applicability in physics.