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Reviewer: Znojil, Miloslav

Reviewer number:

Address:

NPI ASCR, 250 68 Rez,
Czech Republic
znojil@ujf.cas.cz

Author: Nešemán, Jan

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Review text:

Very interesting and compact material (in fact, a PhD thesis) is presented, alas, under a not too informative title. Indeed, all of us might immediately feel discouraged by the fact that there is nothing particularly interesting, say, on the exactly solvable quantum Kepler problem where the interaction potential $V(\vec{r})$ is trivially “unbounded”. Even worse: the author certainly knew many much better titles himself. Unfortunately, he used one of the best-fitting eligible ones (viz., “Relatively bounded perturbations in Krein spaces”) just to give the name to the first chapter of his book. Feeling, presumably, the point in the minimality of the subsequent replacement of “bounded” by “form-bounded” to give the name (and contents) to the second chapter (there are no more chapters in effect, since the third one - in fact, a half-sized last one - just offers illustrative examples).

The book is aimed at, and may be warmly recommended to, all of the mathematicians and/or mathematical physicists who felt, recently, addressed by the upgrade of the formalism of quantum theory carrying, in the current literature, the almost unanimously accepted name which more or less coincides with the first half of the title of the book under review (cf. also the dedicated web-pages <http://ptsymmetry.net/> or <http://gemma.ujf.cas.cz/> Although the young author himself does not seem to have contributed to the (or at least to the here-cited) journal-published literature yet, he was already able to offer a very concise and perfectly mathematically formulated account of the selected bounded-perturbation subfield of the subject. Having built a vocabulary, translation and a bridge between the cultures and languages of the physicists and mathematicians.

It is certainly true that for mathematicians, the subject of the relatively

bounded perturbations of self-adjoint operators in Krein spaces is not a new topic. This is emphasized by this book, in which the mere review of the classical related literature as well as of its recent important perturbation-theory upgrades (cf., in particular, the papers by Caliceti with coauthors between 2005 and 2008) did the very good job. With the upgrade, compactification and completion of the existing spectral-stability results and extension theory in Krein spaces of course - the PhD supervision by Christiane Tretter is well felt behind the scenes.

This being said, it should be added that the physicists working in the field might be disappointed by the scope of the book which provides just a thorough account of the conditions under which the spectrum of the perturbed operators remains real. No account is added of the very recent shift of attention of the community (cf., e.g., the webpage <http://ptqm.physi.uni-heidelberg.de/> of the recent dedicated conference for more details) to the dynamical regime in which the spectrum of the PT-symmetric quantum Hamiltonians degenerates and becomes complex.