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The ultimate purpose of studies like this lies in the constructive build-up of certain new quantum field theories. A few technical aspects of this key idea are tested here via one of the best understood quantum-mechanical models with a non-Hermitian PT-symmetric quantum Hamiltonian $H = (PT)H(PT)^{-1} \neq H^\dagger$ controlled by a repulsive quartic interaction term at large values of coordinates which are (tacitly but very importantly) assumed complex. In a continuation of one of previous papers on this subject (viz., of ref. [5] coauthored by one of the present authors) an innovative insight is gained in several aspects of the puzzle. The $N-$dimensional version of the model is considered in which its $O(N)$ symmetry enables the authors to separate the two-dimensional radial part of Schrödinger equation. In a preparatory step the absence of the so called scattering states is demonstrated in a way paralleling one of proofs proposed, for toy-model potentials $V(x, y) = x^2y^2$, by Simon [11]. Secondly, under the assumption that N is large, a few key features of the ground-state energy are clarified. Variational, Born-Oppenheimer or zero-mass approximations are shown to serve the purpose revealing the presence of a phase transition at certain parameters. The methodical impact of this large$-N$ result is only weakened by the puzzling fact that the domain beyond the phase-transition boundary appears only accessible indirectly, i.e., by means of transition from H to its isospectral Hermitian partner. Indeed, in the light and spirit of the above-mentioned ultimate purpose of the analysis this observation is slightly disappointing because the isospectral-partner operators usually happen to become prohibitively complicated and non-unique in similar physical theories. With the remarkable exception of the particular model in question, brought in attention, implicitly, by Ref. [4] and, explicitly, by Buslaev and Grecchi in J. Phys. A 26 (1993) 5541. Fortunately, one can stay optimistic since, in the context of PT-symmetric theories, the large$-N$ method seems to remain sufficiently robust and efficient not only in all the other similar, exceptionally friendly or almost solvable models [*pars pro toto* let me cite my own large$-N$ approach to anharmonic-Calogero manybody model in J. Phys. A 36 (2003) 9929 plus its PT-symmetry-oriented, Andreas Fring coauthored pendant in J. Phys. A 41 (2008) 194010] but also in many PT symmetric systems with a truly prohibitively complicated structure of their isospectral Hermitian description [in this respect I may finally recommend an illustrative example and more detailed commentary which I quite recently presented in Phys. Lett. A 374 (2010) 807].